**Supporting information**

The supporting information provides the detailed force balance equations used in Pro+ in Section 1 and a user manual for Pro+ in Section 2.

**1. Force balance equations**

The resisting force equations are the same as those used in Yager et al. (2018) except they are rewritten for submerged particles and include a correction for a mistake in equation 4 in Yager et al. (2018). The driving force equations are similar to those used in Kirchner et al. (1990) with modifications to use a different protrusion definition and the velocity profile of Lamb et al. (2017). The effects of channel slope are ignored in the force balance based on the assumption that Pro+ is run for grains that are sitting on a locally flat bed surface. Pro+ is also applied to detrended point clouds. Therefore, we recommend sampling on representative flat bed areas to minimize potential errors with detrending and with using the force balance equations. The force balance equations do not include the effects of turbulence or subsurface flow. Pro+ could be modified to include these effects if desired.

* 1. **Resisting forces**

Details on the derivation of these equations are provided in Yager et al. (2018) and we only provide the equations for reference here. The force due to the submerged weight (*Fg*) of the *ith* grain for the *jth* percentile of the assumed potential pivot angle (*φp*)distribution is

 (1)

where *s* is sediment density, ** is water density, *g* is gravitational acceleration, and *b* is the intermediate grain axis. Two options are available in Pro+ for *φp*: 1) a *φp* of 45° is used, which results in tan(*φp*)=1 and *φp* being effectively removed from the calculations or 2) a distribution of *φp* is calculated using an equation from Kirchner et al. (1990):

*φp*(*i*,*j*)=30+0.5*j*][*b(i)/b50*]0.3 (2)

where *b50* is the calculated median grain size.

The submerged weight (*Fs*) caused by sediment sitting on top of and partially burying the *ith* grain is

(3)

where** is bed porosity and *Vo* is the overlying sediment volume

(4)

where *pR* is the protrusion used to calculate resisting forces. Negative values of *pR* simply mean that the grain is fully buried and any pR<0 is set to *pR*=0. Particles with greater than 100% protrusion (*pR*>*b*) are set to *pR*=*b* because calculated grain volumes cannot exceed those defined by the grain diameter. Similar to equation (1), the tan(*φp*) term in equation (3) is effectively set to one in the calculations if the user specifies to not use *φp*.

The friction force provided by surrounding submerged sediment (*Fsed*) for the *kth* intergranular friction angle (*f*) for each *ith* grain is

(5)

where the buried volume (*Vb*) of the *ith* grain is

. (6)

and *CV* is a volume correction ratio (see Yager et al., 2018). The distribution of *f* is determined using the input mean, standard deviation, and number of samples to take from an assumed normal distribution. The total resisting force (FR) distribution for all grains on the bed is

(7)

**1.2 Driving forces**

The drag and lift forces depend on the flow velocity profile in the roughness layer (equation 11 in Lamb et al., 2017)

(8)

where *z* is the distance from the bed, *u* is the flow velocity, κ is von Karman’s constant, and *ks* is the roughness length. In this equation, *u*(*z*)=0 when *z*=0 and therefore the reference surrounding bed elevation should be a low percentile of the bed elevations surrounding the particle. The lift force (*FL*) follows equation (11) in Kirchner et al. (1990),

(9)

where *A* is the plan view of the cross-sectional area of the grain, *CL* is the lift coefficient, t is the shear stress, and *utop* and *ubot* is the velocity profile evaluated at the top and bottom of the grain, respectively, and

(10)

(11)

where *pD* is the driving protrusion and is defined as the difference between the highest point on a particle and the 10th percentile of the surrounding bed elevation. This effectively assumes that the 10th percentile of the surrounding bed elevation occurs at *z*=0 in the velocity profile equation. The drag force (*FD*) is similar to equation (10) in Kirchner et al. (1990) but with different integration limits,

(12)

where *CL* is the lift coefficient and *Lbot* is lower limit of integration at the bottom of the grain, where

(13)

When *pD*<0, equation (9) results in a zero value for *FL* and equation (12) cannot be solved. Therefore the particle is removed from tc calculations.

**1.3 Critical shear stress calculations**

The sum of *FL* and *FD* is set equal to *FR* and the equations are rearranged to solve for t, which should equal tc at the onset of motion. Critical Shields stresses (t\*c) are calculated as

(14)

The t\*c, *pD*, *pR*, and tc distributions are combined for each grain size bin and for the entire bed.

**2. User guidelines and required inputs/outputs of Pro+**

**2.1 Inputs to Pro+**

Pro+ is coded in matlab through a series of .m function files and requires parameter inputs from an excel file as well as point cloud and grain inputs from a .mat file. As shown in the previous section, the force balance requires several assumed constants, values of φf and φp, as well as *b*, *pD*, and *pR* for every grain. Protrusion (*pD*, and *pR*) for each grain is calculated by Pro+ using the input detrended point cloud and either the input point cloud associated with each grain or the input perimeter associated with each grain from other software. The grain size (*b*) is also input to Pro+ from other software. Pro+ therefore first requires inputs from either: 1) G3Point, which provides the point cloud associated with each grain and grain size, or 2) other software that provides grain perimeter coordinates and grain sizes. The choice between these two input options is provided in the **proinputs.csv** file (Table S1) where the user must specify a value of 1 (use G3Point) or 2 (use other software) for the input variable ‘whichinput’. If G3Point is the chosen input software, then all required output variables from G3Point must be saved in a .mat file that is named within the .csv file (see Table S2). If another pre-run input software is chosen, then all required output variables from that software must be saved in a .mat file that is named within the .csv file (see Table S3).

The **proinputs.csv** file contains all of the other necessary inputs to calculate protrusion and tc in Pro+ (Table S1). Similar to ‘whichinput’ variable above, some input variables require intermediate input variables that are not explicitly part of the force balance equation but provide the user choices about Pro+ calculations of variables. These intermediate inputs in **proinputs.csv** are specified with a ‘–‘ in the first column of Table S1 to denote that although most of these options were discussed in the main text, the specific intermediate input variable was not previously mentioned. We now step through each of the required inputs in Table S1 and their relation to the variables needed to calculate protrusion and tc.

Pro+ calculates the surrounding bed elevation by conducting a circular search with the radius of this search starting at each point on the grain perimeter and being equal to the specified search distance. The locations found in these individual circular searches are then combined for each grain, with repeated locations removed, to create a combined irregular search shape that mimics the particle shape (see Figure 1 in main text). The surrounding bed elevation distribution is determined within this irregular search shape. ‘perimpoints’ allows the user to specify the interval of points on the grain perimeter to employ in the individual point circular search. A larger interval means that fewer points on the grain perimeter will be employed, which results in faster calculations but lower accuracy of the combined irregular search shape. We conducted preliminary tests and found that an optimal interval was every 5 points on the grain perimeter and intervals greater than 10 resulting in potentially low accuracy in the combined irregular search shape.

Pro+ has three choices available for both the search distance (provided in ‘whichradius’) and ks (provided in ‘whichks’): 1) Pro+ calculated sz from the input detrended bed point cloud, 2) Pro+ calculated D84 from the input GSD, or 3) a user specified value. If a value of three is chosen, the user specified value is provided in ‘setradius’ and ‘setks’, respectively. If a value of two or three is chosen, Pro+ automatically calculates the search distance and ks. Unless the user has specific information to provide a better choice of search distance, we suggest testing protrusion and tcvalues with a few potential search distances. In our experiments, protrusion for the finest particles on the bed slightly declined with greater search distance. Smith et al. (2023) also found that for grains smaller than the D50, field measured protrusion also systematically declined with the upstream search distance. Therefore, protrusion and tc for small grains on the bed may be sensitive to the search distance.

For the force calculations, Pro+ has the option to either use no φp (‘usepivot’ set to 0) or use an assumed distribution (‘usepivot’ set to 1) from Kirchner et al. (1990). Pro+ requires the mean, standard deviation, and number of samples within the φf distribution. We recommend using a high number of samples in the φf distribution (e.g., 2000). If one value of f is desired instead of a distribution then that one value is entered as the mean, with a standard deviation of zero, and a sample size of one. Pro+ provides the protrusion and critical shear stress distributions (see next section) for the entire sampled bed area as well as for each grain size bin. The grain size bins can be specified by the user to be either in half phi (-0.5) or whole phi (-1) intervals using ‘phistep’. Finally, values for ρs, ρ, λ, *CD*, *CL*, *Cv*, κ, and *g* must be provided although we strongly recommend using only the default value for *Cv* and default values for κ and *g* are standard.

Pro+ protrusion calculations can be time consuming and are saved as an intermediate step in case using the same values of protrusion are needed while changing the variables that affect tc(e.g., φf, φp, ks). The protrusion values are automatically saved in a file with ‘protrusion’ appended to the provided output filename in ‘outputfile’. This is a separate file from the final protrusion results that are simply saved with the filename provided in ‘outputfile’. If recalculation of tc from the output protrusion is desired, by specifying ‘whichcalcs’ equal to zero. In this case, Pro+ will use the file with the intermediate save protrusion to calculate new values of tc rather than going through all of the protrusion calculations again. The default value of ‘whichcalcs’ is one to specify that all Pro+ calculations will occur including protrusion calculations.

Table S1. Required input variables in the proinputs.csv file.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name in equations/text** | **Name in input file** | **User Options/Explanation** | **Default values** |
| **File to input point cloud/grain sizes from G3Point/other software** | | | |
| **--** | inputfile | Details provided in Tables S2 and S3 | N/A |
| **File to output variables calculated by Pro+** | | | |
| **--** | outputfile | Details provided in Table S4 | ProPlusout.mat |
| Choice in proinputs.csv on which calculations to run | | | |
| **--** | whichcalcs | 0 (do not calculate protrusion, it has already been calculated for these data using a previous run of Pro+), 1 (calculate everything) | 1 (calculate everything) |
| **Choice in proinputs.csv of point cloud/grain size input from G3Point/other software** | | | |
| **--** | whichinput | 1 (use G3Point as input), 2 (use other software as input) | 1 (use G3Point) |
| **Input variables from proinputs.csv used to calculate pD and pR** | | | |
| **--** | whichradius | 1 (search distance=calculated sg), 2 (search distance=calculated D84), 3 (user defined value of search distance) | 3 (user defined value entered in setradius) |
| Search distance | setradius | user defined value in meters when whichradius=3 | 0.014 m (just an example value) |
| -- | perimpoints | any integer value less than or equal to 10 | 5 (recommended value) |
| **Input variables from proinputs.csv used to calculate** tc | | | |
| -- | whichks | 1 (ks=calculated sg) 2 (ks= calculated D84), 3 (user defined value for ks), | 3 (user defined value entered in setks) |
| Roughness length (ks) | setks | user defined value in meters when whichks=3 | 0.014 m (just an example value) |
| -- | usepivot | 0 (do not use φp), 1 (use φp from Kirchner et al. (1990)) | 0 (φp not used) |
| Mean intergranular friction angle (φf) | meanphif | user defined value between 1 and 89° | 60° (just an example value) |
| Standard deviation φf | stdphif | user defined value between 1 and 89° | 15° (just an example value) |
| Number of samples in φf  distribution | numphif | user defined integer value | 2000 (similar high value recommended |
| Sediment density (ρs) | rhos | user defined value in kg/m3 | 2650 kg/m3 (just an example value) |
| Water density (ρ) | rho | user defined value | 1000 kg/m3 (just an example value) |
| Porosity (λ) | porosity | user defined value between 0 and 1 | 0.4 (just an example value) |
| Drag coefficient (CD) | Cd | user defined value | 0.4 (just an example value) |
| Lift coefficient (CL) | Cl | user defined value | 0.2 (just an example value) |
| Volume correction factor (Cv) | Cv | user defined value | 4.827 (recommended value, changing not recommended) |
| von Karman’s constant (κ) | K | user defined value | 0.4 (recommended value) |
| Gravitational acceleration (g) | g | user defined value | 9.81 (changing not recommended) |
| -- | phistep | User defined value in φ units that should be either -0.5 or -1 | -0.5 (recommended value) |
| **Output options in proinputs.csv** | | | |
| -- | saveoutputs | 0 (do not save outputs), 1 (save outputs) | 0 (outputs not saved) |

Table S2. Required inputs if G3Point is used for the grain size and grain point cloud calculations.

|  |  |  |
| --- | --- | --- |
| **Required inputs saved to G3Pointinput.mat** **from G3Point (if whichinput=1)** | | |
| **Name in equations/text** | **Name in G3Pointinput.mat** | **Explanation** |
| point cloud | ptCloud | detrended point cloud from G3Point |
| -- | labels | label to specify part of point cloud associated with each grain |
|  | nlabels | label associated with all identified grains |
| -- | param | used to identify the path |
| -- | Ellipsoidm | used to identify which identified grains were well fit by ellipsoids |
| *--* | granulo | includes grain *b* axes |

Table S3. Required inputs if other software is used for the grain size and grain perimeter calculations.

|  |  |  |
| --- | --- | --- |
| **Required inputs saved to otherinput.mat** **from other prerun software (if whichinput=1)** | | |
| **Name in equations/text** | **Name in otherinput.mat** | **Explanation** |
| point cloud | ptCloud | detrended point cloud. Units must be in meters. |
| grain perimeters | grain.perim | grain perimeters in the same coordinate system as the point cloud. Must be in structure format where fields of grains are in columns and where the field perim provides the grain perimeters. The dimensions of the structure correspond to the number of grains (e.g., grain(1).perim denotes the first grain. Rows in perim denote each point on the grain perimeter with the 1st and 2nd columns corresponding to the *x* and *y* coordinates of each point. Units must be in meters. |
| *b* | b | grain sizes associated with each grain perimeter. Must be organized into a column array. Units must be in meters. |

**2.1 Outputs from Pro+**

‘saveoutputs’ in **proinputs.csv** provides an option to either save (1) or not save (0) Pro+ outputs (Table S4) to a file specified in the csv input file (see Table S1). Pro+ will output the normalized *FR* distribution (*FR*/*Fg*) for the entire sampled bed area, which can be used to calibrate the assumed φf distribution (see main text). The normalized distribution is output to enable comparisons with unsubmerged resisting force measurements using load cells. Pro+ calculated *FR* and *Fg* are for submerged grains whereas most *FR* and *Fg*measurements are for unsubmerged grains. Using the ratio of these forces allows for direct comparisons between submerged and unsubmerged grains. The distributions of *pD*, *pR*, tc, and tc\* are also provided for the entire sampled bed area. Finally, for each ithgrain size bin, Pro+ will output the *pD*, *pR*, tc, and tc\* distributions as well as the representative grain size (Di).

Table S4. Outputs from Pro+

|  |  |
| --- | --- |
| **Outputs from Pro+ to .mat file** | |
| **Name in equations/text** | **Name** |
| Normalized resisting force distribution for entire bed (FR/Fg) | FrWt\_dist |
| Grain size distribution for the entire bed in meters (*b*) | b |
| Driving protrusion distribution for entire bed in meters (pD) | pD |
| Resisting protrusion distribution for entire bed in meters (pR) | pR |
| Critical shear stress distribution for entire bed in Pascals (tc) | taucr\_dist |
| Critical Shields stress distribution for entire bed (tc\*) | taustcr\_dist |
| Grain sizes at the boundaries between each ith bin in meters | bbins |
| Representative grain size for ith bin in meters (Di) | bbinmean |
| Driving protrusion distribution for ith bin in meters | Binned.pRi |
| Resisting protrusion distribution for ith bin in meters | Binned.pDi |
| Critical shear stress distribution for ith bin in Pascals (tci) | Binned.taucri\_dist |
| Critical Shields stress distribution for ith bin (tci\*) | Binned.taustcri\_dist |
| Driving protrusion distribution in the ith bin that corresponds to each critical shear stress value, in meters | Binned.pDi\_for\_taucri |
| Resisting protrusion distribution in the ith bin that corresponds to each critical shear stress value, in meters | Binned.pRi\_for\_taucri |